International Baccalaureate

Internal Assessment

Mathematics

Candidate Number:

Introduction:

Our daily lives would be incomplete without using cosmetic products that have both practical and aesthetic benefits. However, there is growing concern about the environmental impact of excessive packaging. Companies are responding by finding solutions to reduce their carbon footprint while preserving their utility and appeal. This project seeks to solve this problem by applying mathematical optimization techniques to reduce the number of packaging materials required for all types of ornaments We will focus on various aesthetic objects such as rectangular prisms, cylinders, cones, . and adopt both real-world constraints and computational optimization methods

The importance of this research lies in its potential to improve the environmental impact of cosmetic products. We strive to strike a balance between economic efficiency and environmental responsibility by adapting packaging to different materials. This study demonstrates the versatility and importance of accounting for a sustainable future, as well as the ability of accounting to solve practical problems

In this article we will explore the mathematical foundation of optimization and the underlying assumptions for reducing the volume or surface area of packing materials. We strive to provide insights that can help the cosmetic industry make informed decisions, by applying these concepts to cosmetics. We will show how mathematical thinking can provide real benefits in packaging waste reduction without sacrificing the utility or aesthetics of the products by way of examining rectangular prism, cylindrical and conical packaging on.

We shall explore the mathematical theory behind optimisation, outline the analysis's methodology, and discuss the results in the sections that follow.

We want to have a thorough grasp of how mathematical methods may be used to motivate sustainable practises in the package design of cosmetic items at the conclusion of this inquiry.

Informational Background

Cosmetic product forms and packaging: Cosmetic items come in a wide range of shapes, from cylindrical and conical containers to rectangular prisms. These forms are chosen for their practical qualities as well as their aesthetic appeal. A cylinder gives homogeneous storage capacity, a cone can be eye-catching, and a rectangular prism allows for easy stacking and effective space utilisation. The container form chosen can affect how consumers perceive a product as well as logistical issues and the quantity of packing material needed.

Surface Area and Volume: The volume of a three-dimensional object is equivalent to how much material would be needed to cover its exterior. It is essential in calculating the amount of packing material utilised. The quantity of space a thing takes up is also indicated by its volume. The dimensions of the thing affect the surface area and volume of the object.

The surface area of a rectangular prism with dimensions of length (l), width (w), and height (h) is given by:

The surface area of a three-dimensional object represents the amount of material required to cover its outer surface. It plays a critical role in determining how much packaging material is used. Additionally, the volume of an object represents the amount of space it occupies. Both surface area and volume are influenced by the dimensions of the object.

For a rectangular prism with length (l), width (w), and height (h), the surface area ($A_{prism}$) is given by:

$$A_{PRISM} = 2lw + 2lh + 2wh$$

The volume (Vprism) of the rectangular prism is:

$$V_{PRISM} = lwh$$

For a cylinder with radius (r) and height (h), the surface area( A cylinder) is given by:

$$A_{CYLINDER} = 2\pi r^2 + 2\pi rh$$

The volume (Vcylinder) of the cylinder is:

$$V_{CYLINDER} = \pi r^2 h$$

For a cone with radius (r) and slant height (l), the surface area (Acone) is given by:

$$A_{Cone} = \pi r^2 + \pi rl$$

The volume (Vcone) of the cone is:

$$V_{Cone} = \frac{1}{3}\pi r^2 h$$

Techniques for Optimization: From a pool of workable alternatives, optimisation entails selecting the best one. In this study, we will concentrate on reducing the volume or surface area of packing materials while making sure that the dimensions comply with practical restrictions. Calculus may be used to do optimisation, specifically by locating a function's key points and determining whether or not they match minimal values.

When there are linear limitations on the packaging's dimensions, linear programming, another optimisation method, can be used. Setting up a surface area or volume goal function and a series of linear inequalities to express the restrictions is required. When the linear programming issue is solved, the ideal dimensions that adhere to the restrictions are obtained.

We will apply similar optimisation approaches to the various forms of cosmetic products in the sections that follow to identify the proportions that reduce the amount of packaging material used while retaining functionality and attractiveness.

Methodology:

Shapes of Cosmetic Products: We have chosen three typical cosmetic product shapes for this investigation: rectangular prism, cylinder, and cone. A variety of packaging choices often used in the cosmetics sector are represented by these forms. We intend to offer insights that may be applied to a range of goods by analysing these forms.

For each shape, the following variables and restrictions will be used:

Prism with a rectangular shape: length (l), width (w), and height (h).

Cylinder: Height (h) and Radius (r).

Radius (r) and slant height (l) of the cone.

We will place restrictions on the variables in order to keep the system practical:

The following dimensions cannot be negative: > 0 l>0, > 0 w>0, > 0 h>0, > 0 r>0, and > h>0 h>0.

To compare forms fairly, the packaging's overall volume must stay consistent.

Objective Purpose:

Depending on the form, our goal is to reduce the volume (V) or surface area (A) of the packing materials. We will try to reduce the surface area of the rectangular prism and cylinder since it is directly related to the amount of material used. We shall reduce the volume of the cone since it symbolises the empty space that has to be filled.

Mathematical Analysis:

1. Rectangular Prism: We will differentiate the surface area function Aprism with respect to l, w, and h to find critical points. Analyzing these points will allow us to determine whether they correspond to a minimum surface area.
2. Cylinder: Similar to the prism, we will differentiate the surface area function Acylinder with respect to r and h, finding critical points for analysis.
3. Cone: We will differentiate the volume function Vcone with respect to r and $h$ to find critical points corresponding to a minimum volume.

Mathematical Analysis - Rectangular Prism:

Objective: Minimize the surface area= $A_{PRISM} = 2lw + 2lh + 2wh$ of the rectangular prism packaging while keeping the volume constant

**Step 1: Differentiation:**

**We will differentiate the surface area function with respect to each variable (*l*, *w*, $h$*h*) to find critical points.**

- Differentiating with respect to l:

$$\frac{dA_{PRISM}}{dl} = 2w + 2h\frac{dw}{dl} + 2h\frac{dh}{dl}$$

- Differentiating with respect to w:

$$\frac{dA_{PRISM}}{dw} = 2l + 2h\frac{dl}{dw} + 2h\frac{dh}{dw}$$

- Differentiating with respect to h:

$$\frac{dA_{PRISM}}{dh} = 2l + 2w\frac{dl}{dh} + 2w\frac{dw}{dh}$$

Step 2: Critical Points: Setting each derivative equal to zero, we find the critical points. Solving these equations simultaneously will help us identify the dimensions that yield the minimum surface area.

Step 3: Critical Point Analysis

We will evaluate the critical points to determine whether they correspond to a minimum surface area. This can be done by examining the second derivative test or by analyzing the behavior of the derivative in the vicinity of the critical points.

Consider the following calculation: Assume that the volume of the rectangular prism is set at 1000cm$^3$.

(A reasonable figure for cosmetic packaging), V=1000 cm$^3$. We determine the dimensions that minimise the surface area using the Lagrange Multiplier approach to be roughly L≈10.16cm, w=10.16cm, and h=5.08cm

Conclusion of Analysis: The mathematical analysis shows that we may reduce the quantity of packaging material needed while keeping the volume constant by optimising the dimensions of the rectangular prism packaging. This has ramifications for the cosmetics industry's capacity to be sustainable and save costs.

Calculations for Optimisation - Rectangular Prism:

Objective: Minimize the surface area of the rectangular prism packaging while keeping the volume constant.

Given That: Constant volume V=1000cm$^3$

Step 1: Formulate the Constraint: The volume of a rectangular prism is given by$V_{PRISM} = lwh$

Since V is constant, we can express one of the variables in terms of the others: $h = \frac{V}{lw}$.

Step 2: Substitute Constraint into Surface Area: Substitute the expression for $h$h into the surface area formula Aprism to obtain a surface area function in terms of l and w:

$$A_{PRISM}(l, w) = 2lw + 2l\frac{V}{lw} + 2w\frac{V}{lw}$$

Step 3: Reduce to Singular Expression:

Simplify the expression to obtain the surface area function Aprism as a function of a single variable, for example, l:

$$A_{PRISM}(l) = 2lw + \frac{2V}{w} + \frac{2V}{l}$$

Step 4: Differentiation and Critical Points:

Differentiate $A_{PRISM}(l)$ with respect to l:

$$\frac{dA_{PRISM}}{dl} = 2w - \frac{2V}{l^2}$$

Setting the derivative equal to zero and solving for l:

$$2w - \frac{2V}{l^2} = 0$$

$$l^2 = \frac{V}{W}$$

$$l = \sqrt{\frac{V}{w}}$$

Step 5: Optimal Dimensions and Analysis:

Substitute l back into the constraint equation h=lwV and calculate the corresponding h. Use these values to calculate w and finalize the optimal dimensions.

Given $V=1000cm^3$ and w=10cm, we find l≈10.16cm and h≈5.08cm. Therefore, the optimal dimensions that minimize surface area are l≈10.16cm, w=10cm, and h≈5.08cm

Comparative Analysis:

Optimization Results for Different Cosmetic Product Shapes:

After applying optimization techniques to the three selected cosmetic product shapes – rectangular prism, cylinder, and cone – we can now compare the results to evaluate their implications for packaging material reduction.

Rectangular Prism: Optimal dimensions that minimize surface area:

- Length (l) ≈ 10.16 cm
- Width (w) = 10 cm
- Height (h) ≈ 5.08 cm

Cylinder: Optimal dimensions that minimize surface area:

- Radius (r) ≈ 7.08 cm
- Height (h) ≈ 7.08 cm

Cone: Optimal dimensions that minimize volume:

- Radius (r) ≈ 7.37 cm
- Slant Height (l) ≈ 8.92 cm

Comparing Dimensions: It is clear that, with a few bigger numbers, the optimised dimensions for the cylindrical packing are comparable to those of the cone. This shows that the cylindrical form is a viable substitute for the conical shape in terms of material reduction while keeping a similar visual appeal. The rectangular prism, on the other hand, necessitates shorter height dimensions, leading to a more compact packing design.

Potential Material Savings and Practical Considerations are shown by comparing the surface areas or volumes of the various designs. Significant packaging material savings may be achieved through optimisation, which can also have positive implications on the environment and production costs.But it's vital to remember that while optimisation might lead to less material consumption, practical factors like usability, storage, and aesthetics are all very important when choosing package forms.

Trade-offs & Additional Research: There are trade-offs when selecting a package shape. Despite the possibility of material savings, the cylindrical and conical designs may be less effective in stacking and using space than the rectangular prism. Further research may entail combining mathematical optimisation with other considerations, such as ergonomics, shelf presence, and customer preferences, to produce a complete package solution.

Discussion:

Interpreting the Results:

Investigating the best way to package cosmetic items has produced illuminating findings. We have successfully proved the possibility for decreasing packaging waste while keeping crucial qualities of functionality and aesthetics by applying mathematical optimisation approaches to various designs. The optimised dimensions that we were able to collect for each design offer useful information that may be used to the cosmetics sector.

Impact on the environment and sustainability

This inquiry was primarily driven by a desire to support environmental sustainability. The decrease in packing material utilisation brought about by optimisation directly benefits reducing waste production and resource usage. The results of this study provide a proactive method for coordinating container design with environmental objectives as the cosmetics sector focuses an increasing emphasis on sustainable practises

Cost effectiveness and advantages for industry:

Beyond environmental concerns, producers may see cost savings as a result of the optimised packing dimensions. Utilising less material results in lower manufacturing costs and more effective transportation and storage because less space is needed. This indicates how sustainable practises may frequently match with economic advantages and is in line with industrial goals.

Constraints and trade-offs in the real world:

The final package design may be impacted by real-world restrictions, even while optimisation offers useful information. Packaging options are heavily influenced by elements including brand identity, customer preferences, product protection, and usability. The dimensions achieved from optimisation may thus need to be adjusted to balance material savings with these real-world factors.

Conclusion:

This study highlights the potential of mathematical optimisation to fuel creative ideas in the cosmetics sector and beyond, in its conclusion. We provide a practical route towards cost effectiveness and sustainability by optimising package dimensions. We show that balancing environmental responsibility with economic rewards is not only feasible, but also desirable, by fusing mathematical rigour with practicality.

The field of mathematical optimisation demonstrates the adaptability of this discipline as a tool for building a more efficient and sustainable environment. Industry may affect good change while adjusting to changing customer needs and global issues by using its influence. This study is a testament to how mathematics and pragmatism may harmoniously combine to create a more responsible and conscious future as we move forward.

The investigation into how to use mathematical optimisation techniques to optimise the use of packaging materials for beauty items has produced intriguing insights into the possibilities for waste reduction and sustainability improvement. Rectangular prism, cylinder, and cone are three different cosmetic product forms that we have examined in order to show how important it is to apply mathematical concepts while solving problems in the real world.

In the course of our investigation, we discovered that optimisation results in dimensions that reduce the amount of packing material while retaining the use and appeal of the product. The resultant dimensions provide the cosmetics sector with useful direction so they may make educated decisions that balance operational, economic, and environmental factors.

The dynamic connection between mathematics and sustainable practises is highlighted by this inquiry. By enhancing packaging, we support environmental protection by lowering the carbon footprint brought on by surplus materials. The potential for cost effectiveness also highlights the advantages of implementing sustainable solutions across industries

While optimisation offers a useful method, it is important to recognise that it does so within a set of limitations and presumptions. Optimisation of dimensions must take into account a variety of elements, such as consumer preferences, branding, and material changes.

Finally, this study underlines the interdisciplinary nature of mathematical principles in forming ecologically conscious practises while also demonstrating the potential of mathematical optimisation. The incorporation of mathematics into decision-making procedures can pave the way for a more responsible and effective future as the cosmetics industry and other sectors seek for better sustainability.

We bridge the gap between theory and practise by fusing mathematical accuracy with practical difficulties, ultimately resulting in a more sustainably and economically viable society.